

## Exercise 35

Find the critical numbers of the function.

$$g(y) = \frac{y-1}{y^2-y+1}$$

### Solution

A critical number is a value of  $y$  for which the derivative is zero or nonexistent. Take the derivative of the function.

$$\begin{aligned} g'(y) &= \frac{d}{dy} \left( \frac{y-1}{y^2-y+1} \right) \\ &= \frac{\left[ \frac{d}{dy}(y-1) \right] (y^2-y+1) - \left[ \frac{d}{dy}(y^2-y+1) \right] (y-1)}{(y^2-y+1)^2} \\ &= \frac{(1)(y^2-y+1) - (2y-1)(y-1)}{(y^2-y+1)^2} \\ &= \frac{(y^2-y+1) - (2y^2-3y+1)}{(y^2-y+1)^2} \\ &= \frac{-y^2+2y}{(y^2-y+1)^2} \\ &= \frac{-y(y-2)}{(y^2-y+1)^2} \end{aligned}$$

Set what's in the numerator and denominator equal to zero and solve for  $y$ .

$$\begin{aligned} -y(y-2) &= 0 & (y^2-y+1)^2 &= 0 \\ y=0 \quad \text{or} \quad y-2 &= 0 & y^2-y+1 &= 0 \\ & & y &= \frac{1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} \\ & & y &= \frac{1 \pm i\sqrt{3}}{2} \end{aligned}$$

Since only real numbers can be critical numbers of the function,

$$y = 0 \quad \text{or} \quad y = 2.$$